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Reliability-Based Optimization Design of Geosynthetic Reinforced Road Embankment

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16. Abstract

Road embankments are typically large earth structures, the construction of which requires for large amounts of competent fill soil. In order to limit costs, the utilization of geosynthetics in road embankments allows for construction of steep slopes up to 80º - 85º from horizontal, which can save considerable amounts of fill soil in the embankment and usable land at the toe, compared to a traditional unreinforced slope. It then requires for a stability analysis of the geosynthetic-reinforced slope, which is highly dependent on the selection and properties of geosynthetic including tensile strength, transfer efficiency, length and the number of geosynthetic layers placed in embankment, etc. To minimize costs, an optimization design is necessary to select an ideal combination of those design parameters. In this study, reliability-based optimization (RBO) will be implemented on the basis of reliability-based probabilistic slope stability analysis considering the variability of soil properties. RBO intends to minimize the cost involved in geosynthetic reinforced road embankment design while satisfying technical requirements. The limit equilibrium method was embedded to compute the factor of safety (*f*s), meanwhile, the most-probable-point (MPP-) based first-order reliability method (FORM) was conducted to determine the probability of failure (*p*f). The cost is assumed as a function of design parameters: the number of geosynthetic layers, embedment length, and tensile strength of the geosynthetic. Coupling with the reliability assessment and some other technical constraints, the combination of design parameters can be optimized to minimize cost.

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Abstract

Road embankments are typically large earth structures, the construction of which requires for large amounts of competent fill soil. In order to limit costs, the utilization of geosynthetics in road embankments allows for construction of steep slopes up to 80^{o} - 85^{o} from horizontal, which can save considerable amounts of fill soil in the embankment and usable land at the toe, compared to a traditional unreinforced slope. It then requires for a stability analysis of the geosynthetic-reinforced slope, which is highly dependent on the selection and properties of geosynthetic including tensile strength, transfer efficiency, length and the number of geosynthetic layers placed in embankment, etc. To minimize costs, an optimization design is necessary to select an ideal combination of those design parameters. In this study, reliability-based optimization (RBO) will be implemented on the basis of reliability-based probabilistic slope stability analysis considering the variability of soil properties. RBO intends to minimize the cost involved in geosynthetic reinforced road embankment design while satisfying technical requirements. The limit equilibrium method was embedded to compute the factor of safety (f_s), meanwhile, the most-probable-point (MPP-) based first-order reliability method (FORM) was conducted to determine the probability of failure (p_f). The cost is assumed as a function of design parameters: the number of geosynthetic layers, embedment length, and tensile strength of the geosynthetic. Coupling with the reliability assessment and some other technical constraints, the combination of design parameters can be optimized to minimize cost.

Executive Summary

This study examines the optimization design of a geosynthetic reinforced road embankment considering both economic benefits and technical safety requirements. In engineering design, cost is always a big concern. To minimize cost, engineers tend to seek an optimal combination of design parameters among the considered alternatives, while ensuring the optimal design is safe. Reliability-based optimization (RBO) is such a technique that is able to provide engineers the optimal design with the minimum cost while all technical design requirements are satisfied. The idea of RBO is very attractive because of its economic benefits, but so far its application in geotechnical engineering is still very limited and mainly focuses on the design of pile groups and retaining walls. The research goal is to implement mathematical formulation algorithm of RBO in design of geosynthetics reinforced embankment slopes. To achieve this goal, three research objectives have been identified:

- Develop a probabilistic slope stability analysis to assess the reliability of geosynthetics reinforced road embankment;
- Implement reliability-based optimization in design of geosynthetics reinforced road embankment to minimize the cost of geosynthetic reinforcements placed within the slope;
- Perform sensitivity analysis to evaluate the effects of uncertainties in design variables on the reliability and optimal design of geosynthetics reinforced road embankment.

To implement RBO in the design of geosynthetics reinforced embankment system, the stability of a reinforced slope will be studied using limit equilibrium method. Considering geotechnical uncertainties, first-order reliability method (FORM) will be adopted to perform probabilistic slope stability analysis to assess the reliability of the whole system. The system reliability is then used as the crucial constraint in RBO. The constrained optimization problem involved in RBO will be solved by adopting genetic algorithm (GA) so that the optimal design is located. Finally, sensitivity analysis will be carried out to highlight the influence of each design variable on the reliability and optimal design of geosynthetics reinforced road embankment.

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1 Introduction

1.1 Overview

In engineering design, cost is always a big concern. A design should not only be technically feasible, but also economically competent. Usually, there could be various design alternatives to meet the same technical design requirements, but the cost involved could vary significantly. In order to minimize the cost, engineers tend to select an optimal combination of design parameters among the considered alternatives. The process of searching for such an optimal combination is called 'optimization'. In practical design of geotechnical systems, optimization is always performed manually based on the alternatives selected by engineers experience and judgment. However, a crucial issue faced by designers is: when a large number of design parameters are involved, the design process becomes very time consuming and probably fails to find the 'best' optimal result due to the limited number of alternatives the designers can manually try.

In light of the preceding issue, a more systematic and effective optimization approach is required so that the cost of constructed facility is minimized while all technical design requirements are satisfied. Furthermore, due to the unavoidable geotechnical uncertainties, which are primarily arising from inherent soil variability, measurement error and transformation uncertainty (Christian et al. 1995; Phoon & Kulhawy 1999b; Phoon & Kulhawy 1999a; Baecher & Christian 2003), reliability-based analysis has been introduced in geotechnical practice with an intention to assess the risk associated with the design of geo-structures. Therefore, to take the reliability requirements into consideration, reliability-based optimization (RBO) needs to be carried out; wherein the optimization is performed by coupling reliability assessment.

1.1.1 Reliability-based Optimization Design

Theoretically, RBO is a constrained minimization problem; minimizes an objective function while variables are subjected to some reliability constraints. When RBO is applied to the problems of engineering interest, the objective function is always specified as cost function or volume function, while the constraints are determined by design requirements and explicitly model the effects of uncertainties. The idea of RBO is attractive. Substantial studies have been done on solving RBO problems in past decades, as summarized recently in Valdebenito & Schuëller (2010). However, its practical implementation still can be challenging because of the coupling between reliability assessment and cost minimization; the high numerical costs involved in its solution; and the interpretation of a specific engineering problem in mathematical and computational language. So far, the application of RBO in geotechnical engineering is still very limited. Recent studies mainly focus on the design of pile groups (Chan et al. 2009) foundations (Babu & Basha 2008; Basha & Babu 2010) and retaining walls (Babu & Basha 2008; Basha & Babu 2010; Zhang et al. 2011). Few studies have been carried out on the focus of slope design; particularly in the area of reinforced slopes.

As mentioned by Elias et al. (2001), the use of reinforced soil slope (RSS) structures has expanded dramatically in 1990s; approximately 70 to 100 RSS projects were being constructed yearly in connection with transportation related projects in United States, with an estimated projected vertical face area of 130,000 m^2/year. In the last decade, with the developments in reinforcement materials and construction techniques, the use of RSS continuously expands because of its economics and successful performance. Therefore, it can be reasonably expected that great contributions can be made by improving the optimization process in the design of reinforced slopes in practice.

1.1.2 Geosynthetic Reinforced Embankment Slope

Geosynthetic reinforced embankment slope (GRES) is a unique RSS structure which is a form of reinforced soil that incorporates planar geosynthetic reinforcing elements in constructed earth-sloped structures with face inclinations less than 70°; wherein geosynthetics is a generic term that encompasses flexible polymeric materials used in geotechnical engineering (Elias et al. 2001), such as geotextiles, geogrids, geonets, geomembranes, etc.. Among the considered geosynthetics products, geotextiles and geogrids are the two categories used as reinforcement materials most often. A typical GRES system generally consists of foundation, retained backfill, reinforced fill, subsurface drainage, primary reinforcements, secondary reinforcements and surface protection, as shown in Figure 1.1.

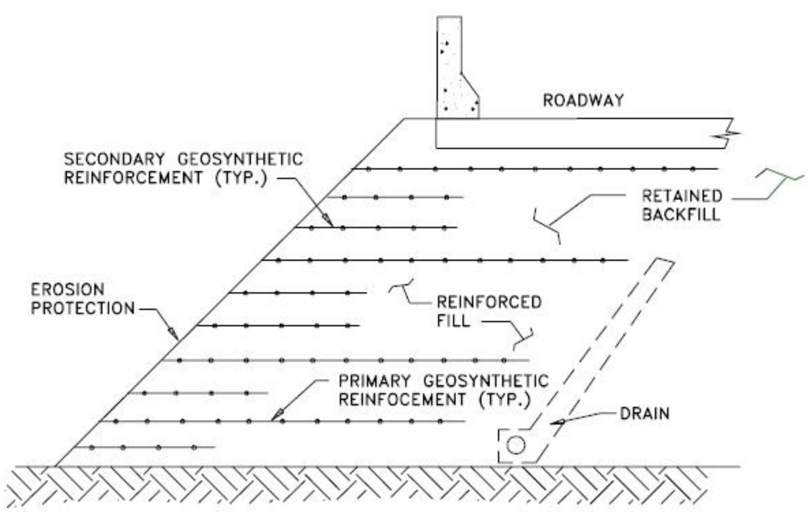

Figure 1.1 Typical components in GRES (Elias et al. 2001)

Primary reinforcements are horizontally placed within the slope to provide tensile forces to resist instability. Either geotextiles or geogrids with sufficient strength and soil compatible modulus can be used as primary reinforcements. Secondary reinforcements are used to locally stabilize the slope face during and after slope construction. In other words, by placing geosynthetic reinforcements, it is able to construct a slope at an angle steeper than could otherwise be safely constructed with the same soil (Elias et al. 2001). Therefore, the use of GRES is able to increase land usage and decrease site development costs. Elias et al. (2001) shows a study of the site-specific costs of soil-reinforced structures based on a survey of state and federal transportation agencies. In general, the use of GRES results in substantial savings about 25 to 50 percent and possibly more in comparison with a conventional reinforced concrete retaining structure, especially when the latter is supported on a deep foundation system. Furthermore, the study provides an approximation of the actual cost of a specific GRES structure, which is basically depending on the cost of each principal component:

- reinforcements: 45 to 65 percent of total cost;
- reinforced fill: 30 to 45 percent of total cost;
- face treatments: 5 to 10 percent of total cost.

The above are the typical relative costs estimated based on limited data. Details may vary with different projects. But basically it concludes the approximate proportions of expenditures, wherein the reinforcement is obviously the principal part, the optimization design of which is expected to be significant to the total cost.

1.2 Objectives

This study is primarily focused on investigating the implementation of RBO in geosynthetic reinforced road embankment design with the intention to minimize the total cost and usage of geosynthetic reinforcements. To achieve this goal, three major research objectives are identified as follows:

- Perform probabilistic slope stability analysis, in which the probability of failure is computed to assess the stability and reliability of geosynthetic reinforced road embankment;

- Develop a RBO framework on the focus of GRES design, wherein the objective function is specified as the cost function with respect to the usage of geosynthetic reinforcements while the crucial constraint is assigned by the previous probabilistic analysis;

- Perform sensitivity analysis to evaluate the effects of the uncertainties in design variables on the reliability and the optimal design of geosynthetic reinforced road embankment.

The proposed framework will allow DOTs to design using a reliability-based procedure that allows the variability of soil properties and geosynthetic inclusions for reinforcement.

2 Stability Analysis for Geosynthetic Reinforced Road Embankment

2.1 Overview

Currently there are three primary methodologies to perform stability analysis for geosynthetic reinforced slopes: Continuum Mechanics, Limit Analysis (LA), and Limit Equilibrium (LE). Continuum mechanics approach is numerically based, such as finite element (FE) or finite difference (FD); considers the full constitutive relationships of all materials involved, e.g. backfills, reinforcements and face treatments. It satisfies boundary conditions, produces displacements (unavailable in LE and LA) and considers local conditions and compatibility between dissimilar materials. Generally, it can represent a problem in the most realistic fashion. To obtain reliable results, it asks for quality input data, which however is frequently not available in common practice. Furthermore, this approach requires a designer with good understanding of possible technical 'traps' during numerical modeling (Christopher et al. 2005; Leshchinsky et al. 2014).

Limit analysis method models the soil as a perfectly plastic material obeying an associated flow rule (Yu et al. 1998). It is able to deal with layered soil, complex geometries, water, seismicity, etc. The numerical upper bound in LA of plasticity yields kinematically admissible failure mechanisms, which means it is not necessary to arbitrarily assume a mechanism as done in LE which is actually an advantage when complex problems are considered (Leshchinsky et al. 2014). However, because of its limited familiarity of practicing engineers, this method is not commonly used in routine design.

Limit equilibrium method has been the most popular method for slope stability calculations by assuming that soil at failure obeys the perfectly plastic Mohr-Coulomb criterion. A major advantage of this approach is its capability to deal with complex soil profiles, seepage and a variety of loading conditions (Yu et al. 1998). As concluded by Christopher et al. (2005), its application to RSS structure is an extension of the classical approach that has been used for unreinforced slopes for decades, that is, investigates the equilibrium of the soil mass tending to slide down under the influence of gravity and surcharge, and evaluates the stability by producing a factor of safety (f_s) which is defined as the ratio of resistance forces (moments) to driving forces (moments) to maintain a static equilibrium. In geosynthetic reinforced slopes, the stabilizing forces contributed by reinforcement layers are incorporated into the limiting equilibrium equations to determine the factor of safety of the reinforced mass. However, unlike the continuum mechanics method, a main concern of this approach is neither LE nor LA considers the compatibility between dissimilar materials. In unreinforced slopes, this issue is always solved by predetermining the failure surfaces according to the prevailing failure mechanism when vastly different soil layers exist. Similarly, in geosynthetic reinforced slope, as mentioned by Leshchinsky et al. (2014), the use of LE in conjunction with soil and geosynthetics is always not much of an issue.

Overall, limit equilibrium method is simple to perform and has been adopted in most of the geotechnical specialized software for slope stability analysis, e.g. Slope/W, Slide, SVSlope, Stable, and some RSS design programs, e.g. ReSSA, MiraSlope, SecueSlope, etc. Furthermore, LE is the method used in ‘’FHWA Mechanically Stabilized Earth Walls and Reinforced Soil Slopes Design & Construction Guidelines" (Elias et al. 2001).

2.2 Limit Equilibrium Method

Substantial studies have been done on the classical limit equilibrium slope stability analysis for unreinforced slopes. Various approaches have been developed based on different failure mechanisms, for example, planar failure analysis is commonly used in the rock masses consisting of planar joints or bedding planes that can be potential planar sliding surfaces; infinite slope analysis is similar to the planar failure analysis but with a sliding surface parallel to the slope face; sliding block method, sometimes, is also called simple wedge method due to the wedge-shaped failure surface; and rotational analysis is always performed on a rotational sliding mass with non-planar failure surface, such as circular or log spiral, which shows to be more common in most of the soil slopes. In geosynthetic reinforced embankment slope, the planar failure (or infinite failure) hardly occurs due to the relatively homogeneous fill material and the localized reinforcements; while the latter two are commonly used in the analyses as demonstrated by Elias et al. (2001).

2.2.1 Sliding block method

For the analysis, the potential sliding block is divided into three parts: an active wedge at the head of the slide; a central block; and a passive wedge at the toe, as shown in Figure 2.1. The factor of safety is computed by summing forces horizontally as given below (Naresh & Edward 2006):

$$f_s = \frac{\text{Horizontal resistance forces}}{\text{Horizontal driving forces}} = \frac{Pp + S}{Pa} \qquad (2.1)$$

Where P_a is active force (driving force), P_p is passive force (resistance force); S is the resistance force due to cohesion of bottom layer, simply $= cL$, wherein c is the cohesion of bottom layer and L is the horizontal width of central block. Several trial locations of the active and passive wedges need to be checked to determine the minimum factor of safety.

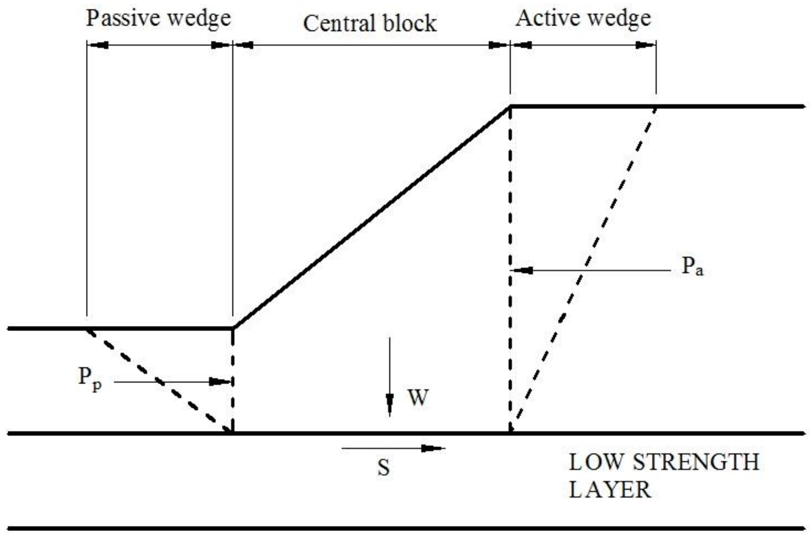

Figure 2.1 Sliding block method

2.2.2 Rotational analysis

During last century, more than 10 methods of slices based on limit equilibrium were developed dealing with circular or arbitrarily shaped rotational slip surfaces (Duncan 1996). Using these methods, a potential slip body is divided into a finite number of vertical slices in order to calculate the forces on each slice, thereby, to determine the factor of safety as follows:

$$f_s = \frac{\text{Resistance moment}}{\text{Driving moment}} \quad (2.2)$$

As concluded by Jiang et al. (2003), the existing methods of slices, e.g., ordinary method, Bishop simplified, Janbu simplified, Spencer, Sarma, and etc., involve various assumptions regarding the interslice forces along with various combinations of equilibrium conditions (force or/and moment) considered, thus giving different values of factor of safety for the same slip surface.

2.2.2.1 Ordinary method

Ordinary method (Fellenius 1936) is the simplest of all with the simplifying assumption that inter slice forces are neglected. This method satisfies only one condition of equilibrium, and is proved to be relatively conservative and underestimates the factor of safety compared to those more accurate methods (e.g., Bishop simplified, Janbu simplified, etc.), that satisfy more than one or more equilibrium conditions. As discussed by Duncan & Wright (1980), its accuracy is good enough for practical purposes in total stress analysis; while the result may be as much as 50% smaller than the ‘correct’ value that is provided by those more accurate methods for flat slopes with high pore pressures in effective stress analysis.

Regardless of the conditioned accuracy, many researchers still use this method, especially in combination with reliability-based analysis (Hassan & Wolff 1999; Xue & Gavin 2007; Ching 2009; Zhang et al. 2009), because of its easy application and computational efficiency.

2.2.2.2 Slip surfaces

The slip surface may vary in different conditions. But in general, a circular failure analysis is sufficient for a slope in a homogeneous soil layer; while for a heterogeneous multi-soil layers slope, a non-circular slip surface seems a better description (Zolfaghari et al. 2005). According to different slip surfaces, the calculation involved in slope stability analysis varies dramatically. Basically, the more complex the surface, the more complicated the calculation is. Therefore, the circular failure analysis is generally the simplest because of the straightforward definition of a circular arc; while an arbitrarily shaped anomalous surface requires more efforts on geometry definition and computational techniques, especially when it is to be combined with the further reliability-based analysis, the difficulties significantly increase.

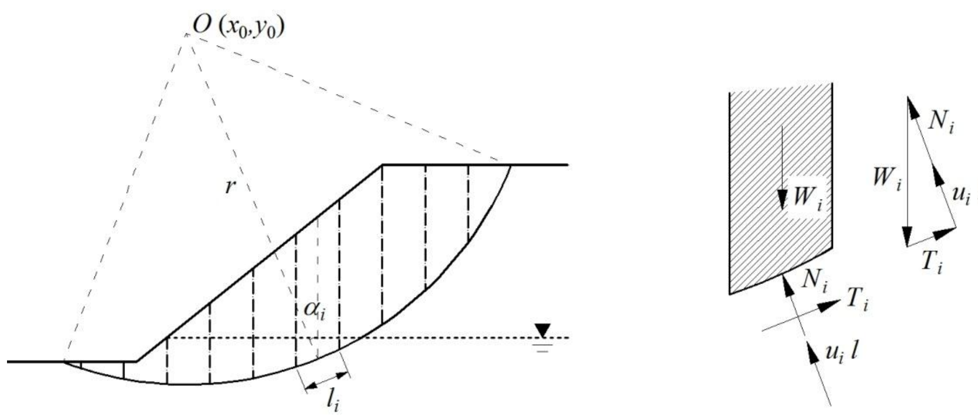

Figure 2.2 The configuration of an unreinforced slope and the forces on a slice with a circular slip surface

2.2.3 Factor of Safety

In the ordinary method, the factor of safety for a circular slip surface in an unreinforced slope (Figure 2.2) is derived based on Equation (2.2), as follows:

$$f_s = \frac{\sum_{i=1}^{n} [c_i' l_i + \tan \phi_i' (W_i \cos \alpha_i - u_i l_i)]}{\sum_{i=1}^{n} W_i \sin \alpha_i} \quad (2.3)$$

where c_i' and ϕ_i' are the effective cohesion and friction angle at the base of the *i*th slice; l_i is the arc length of the slip base of the *i*th slice; W_i is the weight of the *i*th slice; u_i is the porewater pressure acting on the bottom of the *i*th slice; α_i is the tangential inclination on the base of the *i*th slice; and *n* is the number of slices. When the method is implemented in geosynthetic reinforced slope design by adding the contribution of reinforcements directly to the resistance moment, the factor of safety becomes to

$$f_s = \frac{r\sum_{i=1}^{n}\left[c_i' l_i + \tan\phi_i'(W_i\cos\alpha_i - u_i l_i)\right] + \sum_{j=1}^{m} T_j d_j}{r\sum_{i=1}^{n} W_i \sin\alpha_i} \quad (2.4)$$

where T_j is the allowable tensile strength of the *j*th reinforcement layer; d_j is the moment arm of the *j*th reinforcement layer; *r* is the radius of the potential slip surface; and *m* is the number of reinforcement layers, as shown in Figure 2.3. The direction of tensile forces contributed by reinforcement layers and its corresponding moment arm have been the topic of discussion, because the geosynthetic layer is likely to be distorted as rotational deformation occurs. In the limit, the distortion could be orient the geosynthetics along the potential failure arc, thus changing the tensile forces from horizontal direction to tangent direction, and the moment arm from d_j to *r* (Koerner 2005). But in practical design, the horizontal tensile force is preferred because of the more conservative value of d_j.

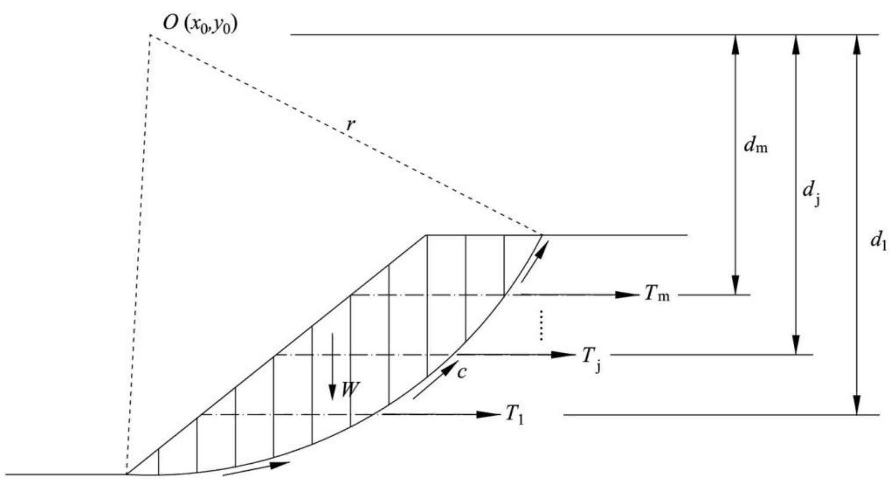

Figure 2.3 The configuration of geosynthetic reinforced embankment and the forces on a circular slip surface

2.3 Reliability-Based Analysis

The uncertainty in slope stability is the result of many factors. Some, such as the ignorance of geological details missed in the exploration program, are difficult to treat formally; others, such as the estimates of soil properties are more amenable to statistical analysis (Christian et al. 1995). As mentioned by Baecher and Christian (2003), the uncertainties in soil properties arise from two primary sources: (1) scatter in data and (2) systematic error in the estimate of the properties. The former consists of inherent spatial variability in properties and random testing errors in their measurement. The latter consists of systematic statistical errors due to the precision of the correlation model used to transform the test result measurement into desired soil property. To take those uncertainties in consideration, reliability-based (or probabilistic) slope stability analysis is carried out. Over the years, a variety of analysis methods have been proposed to perform probabilistic slope stability analysis and a concept of 'probability of failure' is introduced to assess the reliability of the slope system (Cornell 1971; Vanmarcke 1977; Chowdhury & Xu 1994; Christian et al. 1995; Hassan & Wolff 1999; Li & Cheung 2001; Morgenstern & Cruden 2002; Bhattacharya et al. 2003; EI-Ramly et al. 2004; Griffiths & Fenton 2004; Xu & Low 2006; Cho 2007; Ching 2009; Zhang et al. 2011).

2.3.1 Probability of Failure

Mathematically, probability of failure (p_f) is evaluated with the integral as follows:

$$p_f = P\{g(\mathbf{x}) < 0\} = \int_{g(\mathbf{x})<0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \tag{2.5}$$

where $\mathbf{x}$ is the vector of random variables; $g(\mathbf{x})$ is limit state function; $f_x(\mathbf{x})$ is the probability density function (pdf) of random variables. On the basis of reliability theory, probability of failure can be expressed as

$$p_f = 1 - \Phi(\beta) \tag{2.6}$$

where β is reliability index; Φ is cumulative distribution function. When introduced in engineering design, probability of failure is a parameter used to evaluate the impact of uncertainties on the performance of a design, where 'failure' is a generic term for non-performance (Phoon 2008). As in slope stability analysis, it basically means the driving forces (moments) are over the resistance forces (moments) and the static equilibrium state is broken. Thus, the limit state function is always set in form of

$$g(\mathbf{x}) = f_s(\mathbf{x}) - f_{s(\mathbf{r})} \tag{2.7}$$

where $f_{s(r)}$ is the required factor of safety, theoretically set to 1; but may vary with the importance of structures and specific design requirements.

2.3.2 Probabilistic Approach

A number of probabilistic approaches have been proposed to calculate p_f and β. The most popular methods adopted in probabilistic slope stability analysis are first-order second-moment (FOSM), first-order reliability method (FORM), and Monte Carlo simulation (MCS).

Monte Carlo simulation is a sampling-based method, performing random sampling and conducting a large number of experiments on a computer, thus giving conclusions on the model outputs drawn based on statistical experiments. The procedure of MCS is straightforward and most likely to be adopted in the analysis performed using continuum mechanics based method (Morgenstern & Cruden 2002; EI-Ramly et al. 2004; Griffiths & Fenton 2004; Griffiths & Fenton 2007), since which is unable to define a limit state function that is essential to non-sampling probabilistic approaches (e.g. FOSM, FORM). Moreover, because of its high computational costs, MCS is not preferred to be used with limit equilibrium analysis, where considered repetitive analyses are required to seek the critical surface. FOSM and FORM are both non-sampling methods; developed based on a first-order Taylor expansion. In FOSM, the limit state function is approximated with Taylor expansion at the means of random inputs. FOSM is very efficient, convenient and has been adopted in many research works (Chowdhury & Xu 1994; Christian et al. 1995; Hassan & Wolff 1999; Bhattacharya et al. 2003). However, a crucial problem of FOSM is that the method is not invariant; it may change when the limit state function is rearranged to another equivalent form (e.g. Ang & Tang 2007; Zhang et al. 2011). Thereby it becomes quite tricky to decide which form of the limit state function is most appropriate. In light of the invariant issue, FORM is a desired approach in which the first-order approximation is evaluated at a point on the failure surface, thus not influenced by the form of limit state function. In FORM, β can be addressed by solving a constrained optimization problem:

$$\beta = \min_{\mathbf{u}} ||\mathbf{u}||, \text{ sub. to } g^*(\mathbf{u}) = 0 \qquad (2.8)$$

where **u** is a set of independent variables which are derived by transforming the input variables **x** in Equation (2.7) from their original spaces to a standard normal space; $g^*(\mathbf{u})$ is the limit state function in **u**-space (standard normal space). Thereby, from Equation (2.6) the probability of failure can be easily obtained.

The major advantage of FORM is its good balance between accuracy and efficiency: It is invariant compared to FOSM and more efficient compared to MCS especially when probability of failure is low. Therefore, FORM is adopted in many research works (e.g. Low & Tang 1997; Low & Tang 2007; Phoon 2008; Zhang et al. 2011; Cho 2013). But it should be noticed that since the first-order approximation is implemented in both FOSM and FORM, the exact solution is only available when the limit state function is perfectly linear; in nonlinear problems, error arises. In probabilistic slope stability analysis, when Mohr-Coulomb strength parameters are considered as probabilistic variables, from Equation (2.3) and Equation (2.4), it can be noticed ‘tan ϕ’ is the major contributor to the nonlinear performance of the limit state function. With the appropriately selected soil properties, the limit state function is always not too nonlinear, or in other words, close to linear performance. Thereby, many researchers keep using FOSM and FORM in probabilistic slope stability analysis due to their computational efficiency and acceptable accuracy.

2.3.3 Probabilistic Random Variables

In probabilistic slope stability analysis, the Mohr Coulomb strength parameters, cohesion and friction angle, are the two primary random variables that are commonly considered in most of the related studies. The relationship between two random variables often has two possibilities: dependent and independent. Basically, say, there are two random variables, if the occurrence of one does not affect the probability of the other, it is called independence; otherwise, they are dependent. In mathematical way, two independent random variables have the following property: their joint probability distribution is the product of their marginal probability distributions. If the variables are dependent, a measurement parameter, correlation coefficient (Pearson's correlation coefficient) ranging between -1 and 1, is introduced to evaluate the degree of linear dependence between two variables. A positive correlation indicates one tends to go up when another goes up; vice versa, a negative correlation means one tends to go down when another goes up. If the correlation is 1 or -1, the variables are linearly dependent; otherwise, they are non-linearly dependent; while the correlation is zero, the two variables are uncorrelated, but still can be dependent.

Some of the research works assumed independent cohesion and friction angle, which largely simplified the problem (Xue & Gavin 2007; Ching 2009; Zhang et al. 2013), others assumed they are correlated with a nonzero correlation coefficient (Wolff 1985; Chowdhury & Xu 1994; Bhattacharya et al. 2003; Griffiths & Fenton 2004; Zhang et al. 2011). Basically, considering the source of the two strength parameters which are both derived from strength relevant tests, e.g., direct shear test, triaxial test, it is reasonable to believe they are dependent in some pattern; and most likely, negatively correlated, because when a soil has a larger cohesion, the friction angle probably tends to go down to maintain the soil strength within a reasonable range; otherwise, the strength will keep increasing, which is unreasonable and impossible in reality. As discussed by Krounis & Johansson (2011), a reduction in probability of failure of a soil slope was observed as correlation coefficient changes from 1 to -1. Thus, if a negative correlation does exist, the probability of failure can be possibly overestimated by assuming independent parameters; on the other hand, a conservative design is provided. But, after all, the above conclusions are purely observations on some specific examples. The correlation between two or more soil properties is always dependent in varying degrees on soil type, testing method used to obtain the numerical value of the parameter, and the homogeneity of the soil (Uzielli 2007). If there are enough data, the correlation may able to be interpreted based on probability theory as follows:

$$\rho_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{(x_i - \bar{x})^2}\sqrt{(y_i - \bar{y})^2}} \quad (2.9)$$

where ρ_s is the sample Pearson correlation coefficient; otherwise, assumption has to be made based on the previous investigations and works, or those published correlation models. But in light of the site-specific characteristics, inappropriate assumption may arise underestimate or overestimate in results that needs to be kept in mind.

2.3.4 MPP-Based FORM

FORM is developed on the basis of the first-order Taylor expansion which is evaluated at a point on the failure surface, the shortest distance from which to the origin is defined as the reliability index (β); afterwards, the probability of failure can be computed according to Equation (2.6). Thereby, the problem can be easily solved once it is able to locate the most probable point, $\mathbf{u}^*$, which is the shortest distance point from the origin to the limit state curve $g(\mathbf{u})$ in Figure 2.4, following the search algorithm as demonstrated in Figure 2.5 to address a minimization problem with an equality constraint as described in Equation (2.8).

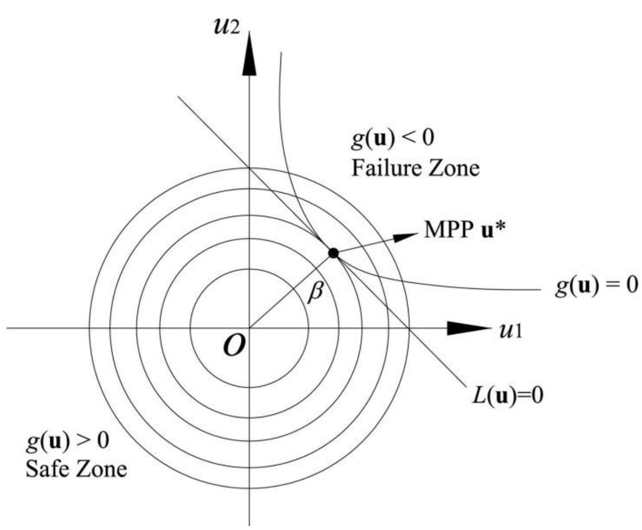

Figure 2.4 Probabilty integration in a two-dimensional standard normal space in FORM

Prior to searching for the most probable point, the random variables need to be transformed from their original random space into a nondimensional, standard normal space (**u**-space in Figure 2.4). When the variables are independent, Rosenblatt transformation can be applied as follows:

$$u_i = \Phi^{-1}\left[F_i\left(x_i\right)\right] \quad (2.10)$$

where $F_i(\cdot)$ is the cumulative distribution function of the variable x_i. If the variables are correlated, the transformation becomes more complicated during which Cholesky decomposition needs to be introduced to decompose the correlation matrix, thereby, transform the correlated variables into independent ones.

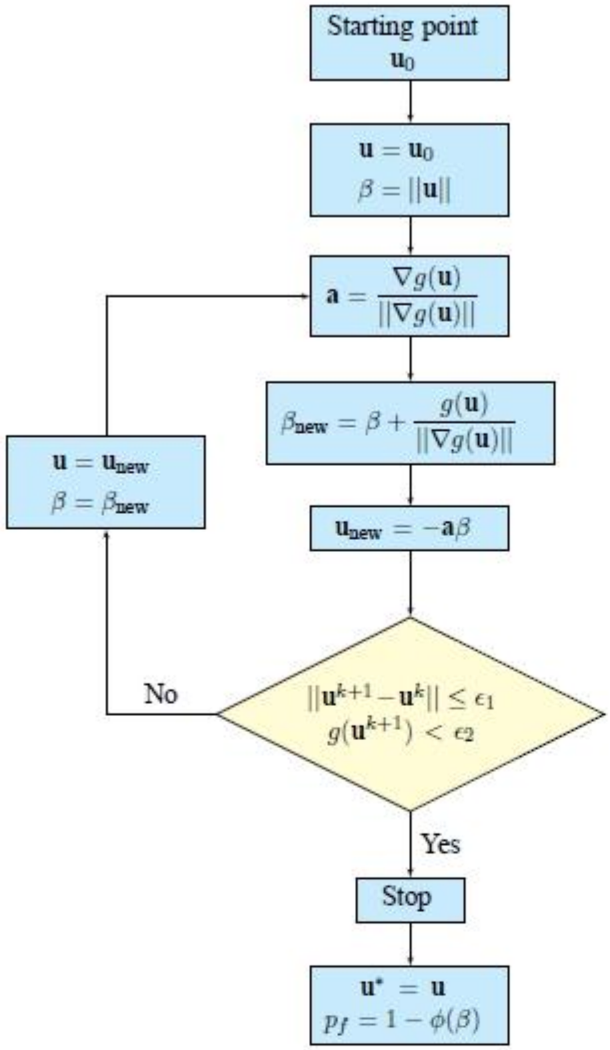

Figure 2.5 Search algorithm for locating MPP

The critical equilibrium for steep reinforced slopes is usually governed by long-term stability conditions. Therefore, in probabilistic slope stability analysis, the effective strength parameters, cohesion (c') and friction angle (ϕ') are considered as probabilistic random variables, along with the allowable tensile strength of geosynthetic reinforcements (T_a) in this study. Therefore, we have

$$\mathbf{x} = (c', \phi', T_a).$$

According to Equation (2.3), (2.4) and (2.7), the limit state functions are derived as

$$g(\mathbf{x}) = f_s(\mathbf{x}) - 1 = \frac{\sum_{i=1}^{n} [c_i' l_i + \tan\phi_i'(W_i \cos\alpha_i - u_i l_i)]}{\sum_{i=1}^{n} W_i \sin\alpha_i} - 1 \qquad (2.11)$$

$$g(\mathbf{x}) = f_s(\mathbf{x}) - 1 = \frac{r\sum_{i=1}^{n} [c_i' l_i + \tan\phi_i'(W_i \cos\alpha_i - u_i l_i)] + \sum_{j=1}^{m} T_j d_j}{r\sum_{i=1}^{n} W_i \sin\alpha_i} - 1 \qquad (2.12)$$

where Equation (2.11) refers to unreinforced slopes, and Equation (2.12) is for geosynthetic reinforced slopes. Then, following the MPP search procedure as shown in Figure 2.5, the probability of failure of the slope system can be computed.

2.4 Critical Slip Surfaces

In slope stability analysis, it is routine to search for a slip surface along which the slope is most likely to fail; in other words, the most dangerous surface (or critical slip surface).

2.4.1 Deterministic Analysis

Conventionally, all the design parameters, e.g., Mohr-Coulomb strength parameters and tensile strength of geosynthetic reinforcement, are deterministic. The conventional analysis is accordingly 'deterministic' as well and requires many analyses of different potential slip surfaces in order to arrive at the surface with the lowest factor of safety, which is called 'critical deterministic surface'. The problem of locating this surface is formulated as an optimization problem (Li & Cheung 2001):

$$\min_{\text{surface}} f_s\left(\mathbf{p}, x_1^{(k)}, y_1^{(k)}, x_2^{(k)}, y_2^{(k)}, \cdots\right) \qquad (2.13)$$

where **p** is the collection of input geotechnical parameters; $\{x_1^{(k)}, y_1^{(k)}, x_2^{(k)}, y_2^{(k)}, \ldots\}$ is the set of shape variables (location parameters) defining the location of slip surface for kth trial; f_s is the factor of safety for a given set of geotechnical parameters and a given geometry of slip surface defined by location parameters. It is a general form dealing with any shaped surfaces. In a more specific way, for a circular slip surface, there are only three shape variables: x and y ordinates of the center of rotation and the radius of slip surface. Then the problem stated in Equation (2.13) can be simplified as follows:

$$\min_{\text{surface}} f_s\left(\mathbf{p}, x_0^{(k)}, y_0^{(k)}, r^{(k)}\right) \quad (2.14)$$

where $\{x_0^{(k)}, y_0^{(k)}\}$ is the center of rotation for *k*th trail; $r^{(k)}$ is the radius of the slip surface for *k*th trail.

2.4.2 Probabilistic Analysis

Similar to the deterministic analysis, probabilistic analysis tends to address the surface with the highest probability of failure (or the lowest reliability index). Such a surface is called `critical probabilistic surface'. The search form is not different in concept from that of critical deterministic surface, and can be formulated in exactly the same way as above (Li and Cheung 2001). Generally, the problem is stated as

$$\max_{\text{surface}} p_f\left(\mathbf{p}, x_1^{(k)}, y_1^{(k)}, x_2^{(k)}, y_2^{(k)}, \cdots\right) \quad (2.15)$$

For a circular slip surface, it is

$$\max_{\text{surface}} p_f\left(\mathbf{p}, x_0^{(k)}, y_0^{(k)}, r^{(k)}\right) \quad (2.16)$$

where p_f is the probability of failure for a given set of geotechnical parameters and a given geometry of the slip surface defined by location parameters.

2.4.3 Search Approach

The critical deterministic and probabilistic surfaces can be located by solving the optimization problems as stated in Equation (2.13), (2.14), (2.15) and (2.16). For a circular slip surface, the most commonly used method is Grid-line search method, in which a predetermined set of grid lines is set for possible locations of the center of slip circle. All the nodal points defined by grid lines are searched to locate those two critical surfaces with different radii. Grid-line method is simple to implement and is embedded in most of the commercial slope stability programs. Otherwise, a variety of search methodologies are proposed, including: the classical methods, such as the alternating variable technique (Li & Lumb 1987), simplex method (Nguyen 1985; Chen & Shao 1988), conjugate-gradient method (Arai & Tagyo 1985), dynamic programming (Yamagami & Jiang 1997); Monte Carlo technique (Greco 1996); and more recently, the heuristic algorithms, such as simulated annealing algorithm (Cheng 2003; Su 2008), genetic algorithm (McCombie & Wilkinson 2002; Cheng 2003; Zolfaghari et al. 2005; Xue & Gavin 2007; Sengupta & Upadhyay 2009; Talebizadeh et al. 2011) and etc.. But most likely, they are used for non-circular surfaces, the number of the location parameters of which is usually much greater than three (for circular surface). Thus, the geometric method, such as grid-line method, becomes inefficient and requires a lot of effort in defining the solution domain for each location parameter (Phoon 2008).

As discussed by Hassan and Wolff (1999), the critical deterministic and probabilistic surfaces may be located at different positions. But Li and Lumb (1987) emphasized the observation that those two surfaces are very close to each other for homogeneous natural slopes, thus proposed that the location of critical deterministic surface could be used as a starting location for searching for the critical probabilistic surface. However, as the writers said, this is purely an observation, not universally true; and it is only for unreinforced slopes. As for reinforced slopes, few studies were carried out with such a discussion. Therefore, it is more reasonable to perform a simultaneous search, as stated by Bhattacharya et al. (2003) and Xue and Gavin (2007), for reinforced slopes; which is exactly embedded in this study.

3 Reliability-Based Optimization Design

3.1 Overview

Reliability-based optimization allows determining the best designs solution (with respect to prescribed criteria) while explicitly considering the unavoidable effects of uncertainty. In general, the application of RBO is numerically involved, as it implies the simultaneous solution of an optimization problem and also the use of specialized algorithm for quantifying the effects of uncertainties (Valdebenito & Schuëller 2010). A typical formulation of RBO is given by

$$\begin{aligned} &\min f(\mathbf{d}, \mathbf{X}, \mathbf{P}) \\ &\text{sub. to}: P\{g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leq 0\} \leq p_{f_i},\ i = 1, 2, \cdots, m \end{aligned} \quad (3.1)$$

where f is the objective function; **d** is the set of deterministic design variables; **X** is the set of random design variables; **P** is the vector of random design parameters; $g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})$ are constraint functions; p_{fi} are desired probabilities of constraint satisfaction; and m is the number of probabilistic constraints. The elements in vector **d** and **X** are the design variables that need to be determined through optimization.

3.2 Optimization Design for Geosynthetics Reinforced Road Embankment

The goal of this phase is to minimize the cost of geosynthetic reinforcements considering potential failure possibilities of a geosynthetic reinforced road embankment. The objective function is specified as the total cost with respective to the usage of geosynthetic reinforcements, as stated by

$$f(n_r, \mu_T, \mathbf{P})$$

where f is the total cost function; n_r is the number of reinforcement layers; μ_T is the mean of tensile strength of geosynthetic reinforcements; and **P** is the vector of the rest design parameters = (cov_T, $\mu_{c'}$, $\sigma_{c'}$, $\mu_{\phi'}$, $\sigma_{\phi'}$). Therefore, the optimization problem can be described as

$$f(n_r, \mu_T, \mathbf{P}) = \text{Cost or Usage}$$

$$\text{subject to: 1. } P\{g_i(n_r, \mu_T, \mathbf{P}) < 0\} \leq p_{fi}$$

$$\text{2. } n \in [n_l, n_u], \text{interger; and } \mu_T \in [\mu_{Tl}, \mu_{Tu}]$$

where n is the number of layers; μ_T is the mean of allowable tensile strength of geosynthetics; n_l, n_u, μ_{Tl}, μ_{Tu} are the lower and upper bounds for n and μ_T respectively.

3.2.1 Usage Function

The cost is primarily depending on the usage and unit price of each component in reinforced slope system. The definition of the objective function is expected to influence the optimal results significantly. As mentioned in section 1.1.2, the reinforcing elements in a geosynthetic reinforced embankment slope commonly consist of primary reinforcements and secondary reinforcements; wherein the usage of primary reinforcements can be computed as

$$L_p = \sum_{j=1}^{n_r} L_{\text{total}}(j) = \sum_{j=1}^{n_r} [L_e(j) + L_{\text{slip}}(j)] = n_r \frac{f_{s(pl)}}{2\tau E}\mu_T + \sum_{j=1}^{n_r} L_{\text{slip}}(j) \tag{3.2}$$

where L_e is anchorage length; t is the shear stresses along geotextile surfaces (assumed as uniformly distributed along geotextile); E is the transfer efficiency of geotextile; $f_{s(pl)}$ is the required factor of safety of geotextile; the length within the slip body, L_{slip}, can be simply determined by taking the distance from the slope face to the failure plane. The usage of secondary reinforcements (L_s) is highly depending on construction regulations, such as:

- The secondary reinforcements must be installed when the spacing of primary reinforcements is over 60 cm;
- The spacing of secondary reinforcements is typically 30 to 50 cm;
- The embedment length of secondary reinforcements is typically 1.0 to 1.5 m.

3.2.2 Cost Function

Basically, the total cost is the product of the reinforcing elements usage and the corresponding unit prices; wherein the usage of reinforcements is mainly controlled by design variables and can be straightforwardly described as shown in section 3.2.1, while the unit price of the products needs to be provided by manufactures or the geosynthetic companies. Generally, the unit price varies with products properties and performances.

3.3 Optimization Approach

The most direct approach for solving a RBO problem is implementing a double-loop strategy (Valdebenito & Schuëller 2010), the formulation of which is stated in Equation (3.1). It employs nested optimization loops as shown in Figure 3.1 to first evaluate the reliability of each probabilistic constraint (inner loop) and then to optimize the design objective function subject to the reliability requirements (outer loop) (Reddy et al. 1994; Wang et al. 1995; Tu et al. 1999). Because of its easy application, double-loop strategy is implemented in most of the research work regarding the reliability-based optimization design in geotechnical engineering (Wang & Kulhawy 2008; Chan et al. 2009; Wang 2009; Talebizadeh et al. 2011; Zhang et al. 2011). Otherwise, to improve the efficiency of double-loop strategy, some other techniques were introduced such as to improve the efficiency of uncertainty analysis, e.g., the methods of fast probability integration (Wu 1994), two-point adaptive nonlinear approximations (Grandhi & Wang 1998); or to modify the formulation of probabilistic constraints, e.g., single-loop (Chen & Hasselman 1997), decoupling approach (Li & Yang 1994).

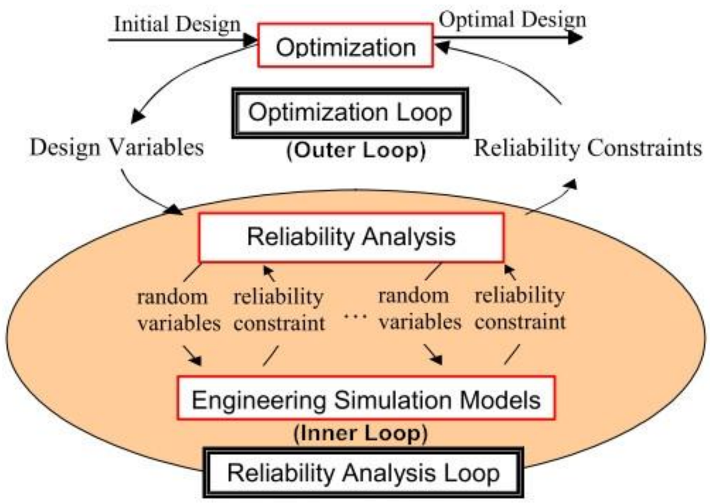

Figure 3.1 A double-loop procedure, adapted from Du et al. 2007

But no matter which strategy is employed, the optimization task is always involved, that is, minimizes the objective function subject to the constraints. Such a constrained optimization problem can be solved by implementing the methods as mentioned in section 2.3.3. However, different from searching for the critical deterministic and probabilistic surfaces that usually come with continuous objective functions and smooth constraints, the optimization design of geosynthetic reinforcements includes non-smooth constraints, e.g. the number of layers should be integer. Therefore, heuristic algorithms, e.g. simulated annealing algorithm and genetic algorithm, are preferred in design of geo-structures rather than the classical methods (Wang and Kulhawy 2008; Chan et al. 2009; Wang 2009; Talebizadeh et al. 2011; Zhang et al. 2011b).

4 Sensitivity Analysis

4.1 Overview

The significance of random variables on the probability of failure of slopes is generally evaluated by changing different set of values for each variable and repeating the approach several times to conclude a trend. This conventional method is very straightforward, but requires time and computational resources. Especially when the number of random variables is large, it is inefficient to change the distributions of all the random variables to get a conclusion. This section presents an analysis approach in which the sensitivity analysis is introduced to quantify the influence of the random variables on the probability of failure for slopes.

4.2 MPP-Based Probabilistic Sensitivity Analysis

The sensitivity analysis is conducted based on MPP-based FORM in order to quantify the impact of uncertainties in random variables on the uncertainty in model outputs, e.g. the probability of failure of the slope system. The significance can be specified by a probability-based sensitivity measure, which is defined as the rate of the change in probability of failure due to the change in a distribution parameter p of random variable, x_i, as (Guo & Du 2009):

$$s_p = \frac{\partial p_f}{\partial p} \quad (4.1)$$

After a series of transformation, the sensitivity can be calculated by

$$s_p = \frac{\partial p_f}{\partial p} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial p} \quad (4.2)$$

where $w(p) = \Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right]$.

For normally independently distributed random variables, since

$$w\left(\mu_{x_i}, \sigma_{x_i}\right) = \Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right] = \Phi^{-1}\left[\Phi\left(\frac{x_i - \mu_{x_i}}{\sigma_{x_i}}\right)\right] = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad (4.3)$$

the sensitivity measures with respect to the mean and standard deviation of random variable, x_i, are given by (Guo and Du 2009)

$$s_{\mu_i} = \frac{\partial p_f}{\partial \mu_{x_i}} = -\phi(-\beta)\frac{u_i^*}{\beta \sigma_{x_i}} \tag{4.4}$$

$$s_{\sigma_i} = \frac{\partial p_f}{\partial \sigma_{x_i}} = -\phi(-\beta)\frac{\left(u_i^*\right)^2}{\beta \sigma_{x_i}} \tag{4.5}$$

For log-normally independently distributed random variables, $x_i \sim \ln(\mu_{xi}, \sigma_{xi})$, the sensitivity measures are given by

$$s_{\mu_i} = \frac{\partial p_f}{\partial \mu_{x_i}} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \mu_{x_i}} \tag{4.6}$$

$$s_{\sigma_i} = \frac{\partial p_f}{\partial \sigma_{x_i}} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \sigma_{x_i}} \tag{4.7}$$

$$\begin{aligned} w\left(\mu_{x_i}, \sigma_{x_i}\right) &= \Phi^{-1}\left\{F_{\ln x_i}\left[\ln x_i^*\right]\right\} = \Phi^{-1}\left[\Phi\left(\frac{\ln x_i^* - \mu_{\ln x_i}}{\sigma_{\ln x_i}}\right)\right] \\ &= \frac{\ln x_i^* - \mu_{\ln x_i}\left(\mu_{x_i}, \sigma_{x_i}\right)}{\sigma_{\ln x_i}\left(\mu_{x_i}, \sigma_{x_i}\right)} \end{aligned} \tag{4.8}$$

where $u_i^* = \frac{\ln x_i^* - \mu_{\ln x_i}}{\sigma_{\ln x_i}}$. If the random variables are correlated, the derivations will be much more complicated, wherein Cholesky decomposition needs to be embedded.

5 Conclusions

This study provides a framework of how to implement the reliability-based optimization in the design of geosynthetic reinforced road embankment. Compared to the conventional way that seeks an optimal design by manually repeating the design process based on the alternatives selected based on engineers experience and judgments, the proposed framework is more systematic and effective and allows DOTs to design using a reliability-based procedure that allows the variability of soil properties and geosynthetic inclusions for reinforcement. The framework is summarized by the flowchart as shown in Figure 5.1. All the codes are written in Matlab, which are attached in Appendix.

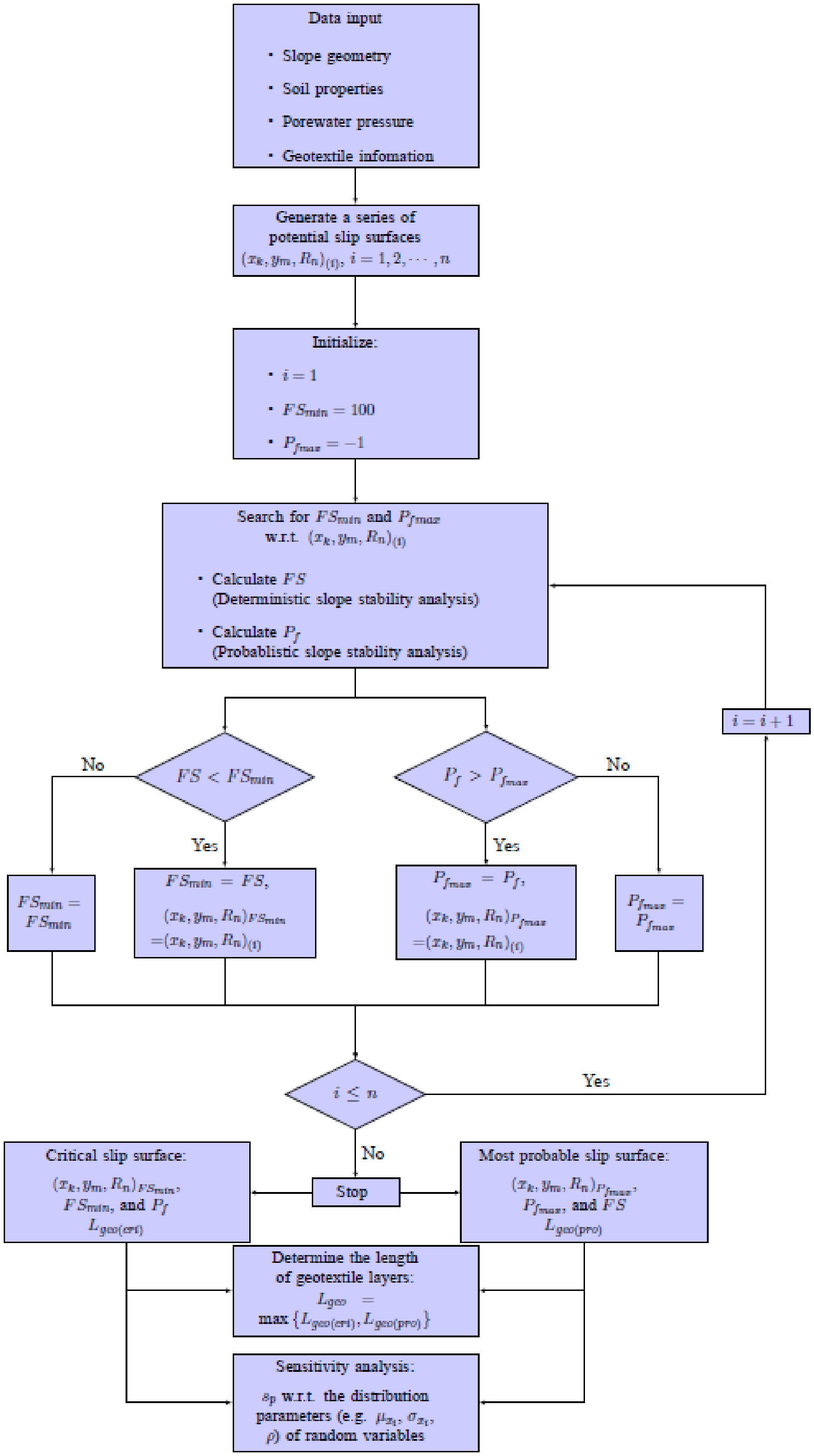

Figure 5.1 Design flowchart of RBO for geosynthetic reinforced road embankment

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Appendix

```
% ----------Reliability-based Probabilistic Slope Stability Analysis-------
%                    Author: Mingyan Deng
%                  Mod. Date: April 11, 2013
%                Slip surface: Circular
%                Slice method: Ordinary method
%                Pro Analysis: MPP-based FORM
%               Reinforcement: Geotextiles
% ------------------------------------------------------------------------
% -----------------------------Datebase----------------------------------
% geometry information
gdata    = Geometry_Slope;
x1       = gdata(1);
y1       = gdata(2);
H        = gdata(3);
aSL      = gdata(4);            % slope angle in degree
apSL     = aSL*pi/180;          % slope angle in pi
B1       = H/tan(apSL);         % width of slope
x2       = x1+B1;               % x coordinate of top
y2       = y1+H;                % y coordinate of top

% soil property information
sdata     = Soil_Property;
ce        = sdata(1);
std_ce    = sdata(2);
tfrie     = sdata(3);
std_tfrie = sdata(4);
unitwW    = sdata(5);
unitwS    = sdata(6);
nslice    = sdata(7);
dis_ce    = sdata(8);
dis_tfrie = sdata(9);
rho_cf    = sdata(10);

% grids of potential slip surface information
grdata   = Grids_Define;
nx0      = grdata(1);              % number of nodes in x-direction
ny0      = grdata(2);              % number of nodes in y-direction
x0_max   = grdata(3);              % range of grids in x-direction
x0_min   = grdata(4);
y0_max   = grdata(5);              % range of grids in y-direction
y0_min   = grdata(6);
nR       = grdata(7);              % number of radius of potential slip surface
R_max    = grdata(8);              % range of radius
R_min    = grdata(9);
inv_x0   = (x0_max-x0_min)/(nx0-1);   % grids interval in x-direction
```

```
inv_y0   = (y0_max-y0_min)/(ny0-1);   % grids interval in y-direction
inv_R    = (R_max-R_min)/(nR-1);      % interval for radius
x0       = x0_min:inv_x0:x0_max;      % grids nodes in x-direction
y0       = y0_min:inv_y0:y0_max;      % grids nodes in y-direction
R        = R_min:inv_R:R_max;         % potential radius
ntrial   = nx0*ny0*nR;                % number of iterations in searching for c.s.s.

% porewater pressure information
u = Porewater_Define;                 % define porewater pressure

% geotextile information
geotextile_data      = Geotextile_information;
geo_layer_y1         = geotextile_data(1);
geo_layer_yn         = geotextile_data(2);
number_of_layers     = geotextile_data(3);
tension_allowable    = geotextile_data(4);
required_FS          = geotextile_data(5);
transfer_efficiency  = geotextile_data(6);
dis_geotextile       = geotextile_data(7);         % distribution type of allowable tension of geotextile
dis_geo_parameter_1  = geotextile_data(8);         % first distribution parameter: if it is normal or
lognormal distribution, it'll be mean value; if it is uniform distribution, it'll be upper bound
dis_geo_parameter_2  = geotextile_data(9);         % second distribution parameter: if it is normal or
lognormal distribution, it'll be std value; if it is uniform distribution, it'll be lower bound
y_geotextile         = y_location_geotextile(geo_layer_y1,geo_layer_yn,number_of_layers);

% ---------------------------Initialization-------------------------------
FS_initial = 100;
Pf_initial = -1;
% ----------Search for critical and most probable failure surfaces---------
for k = 1:nx0
   for m = 1:ny0
      for n = 1:nR

         % Define potential slip surfaces
         [xs_max,xs_min]   = Slip_Surface_Define(x1,y1,apSL,x0(k),y0(m),R(n));
         [xs1,ys1,xs2,ys2] = Cases(xs_max,xs_min,x1,y1,x2,y2,apSL,x0(k),y0(m),R(n));

         % Plot potential slip surfaces
         [xa,ya] = Plot_Potential_Slip_Surface(x0(k),y0(m),R(n),xs1,xs2);
%           plot(xa,ya); hold on;

         % Define parameters
         Slength      = sqrt((xs1-xs2).^2+(ys1-ys2).^2);
         Angle_Center = 2*asin(Slength./(2*R(n)))*180/pi;
         ArcLength    = 2*pi*R(n).*Angle_Center/360;       % arc length for potential slip surface
         xlength      = (xs2-xs1)./nslice;                 % slice width in x-direction

% --------------------Searching for critical slip surface------------------
```

```
        ORD_result =
Search_for_MinFS_reinforcement(x1,x2,y1,y2,apSL,ce,tfrie,unitwS,nslice,xs1,xlength,ArcLength,x0(k),y0(
m),R(n),y_geotextile,number_of_layers,tension_allowable);
            HF = ORD_result(1);
            Ne = ORD_result(2);
        FS_current = ORD_result(3);
        if FS_current < FS_initial
          FS_initial = FS_current;
          n_FS_min   = n;
          m_FS_min   = m;
          k_FS_min   = k;
          HF_FS_min  = HF;
          Ne_FS_min  = Ne;
          AL_FS_min  = ArcLength;
          xl_FS_min  = xlength;
          xs1_FS_min = xs1;
          ys1_FS_min = ys1;
          xs2_FS_min = xs2;
          ys2_FS_min = ys2;
        else
          FS_initial = FS_initial;
        end
        FS_min = FS_initial;

% ----------------Searching for most probable failure surface--------------

        MPP_result = MPP_cor('g_function_MPP_rein','partial_g_u_rein',[dis_ce dis_tfrie
dis_geotextile],[rho_cf 0 0],[ce tfrie dis_geo_parameter_1],[std_ce std_tfrie
dis_geo_parameter_2],[ArcLength HF Ne R(n) geo_layer_y1 y0(m) geo_layer_yn number_of_layers]);
        Uc_current   = MPP_result(1);
        Uf_current   = MPP_result(2);
        Ug_current   = MPP_result(3);
        Beta_current = MPP_result(4);
        Pf_current   = MPP_result(5);
        if Pf_current > Pf_initial
          Pf_initial  = Pf_current;
          MinBeta     = Beta_current;
          Uc_MaxPf    = Uc_current;
          Uf_MaxPf    = Uf_current;
          Ug_MaxPf    = Ug_current;
          n_Pf_max    = n;
          m_Pf_max    = m;
          k_Pf_max    = k;
          HF_Pf_max   = HF;
          Ne_Pf_max   = Ne;
          AL_Pf_max   = ArcLength;
          xl_Pf_max   = xlength;
```

```
            xs1_Pf_max  = xs1;
            ys1_Pf_max  = ys1;
            xs2_Pf_max  = xs2;
            ys2_Pf_max  = ys2;
        else
            Pf_initial = Pf_initial;
        end
        Pf_max = Pf_initial;
      end
   end
end

% ----------------------Along critical slip surface------------------------
MinFS      = FS_min;                  % minimum factor of safety
MinFS_x0   = k_FS_min;                 % iteration step of x-center
MinFS_y0   = m_FS_min;                 % iteration step of y-center
MinFS_R    = n_FS_min;                 % iteration step of radius
x0_cri     = x0(MinFS_x0);             % x of center of slip surface
y0_cri     = y0(MinFS_y0);             % y of center of slip surface
R_cri      = R(MinFS_R);               % radius of slip surface
location_c = [x0_cri y0_cri R_cri];

% probability of failure along critical slip surface
MPP_MinFS  = MPP_cor('g_function_MPP_rein','partial_g_u_rein',[dis_ce dis_tfrie
dis_geotextile],[rho_cf 0 0],[ce tfrie dis_geo_parameter_1],[std_ce std_tfrie
dis_geo_parameter_2],[AL_FS_min HF_FS_min Ne_FS_min R_cri geo_layer_y1 y0_cri geo_layer_yn
number_of_layers]);
Uc_MinFS   = MPP_MinFS(1);
Uf_MinFS   = MPP_MinFS(2);
Ug_MinFS   = MPP_MinFS(3);
Beta_MinFS = MPP_MinFS(4);
Pro_MinFS  = MPP_MinFS(5);

% required geotextile length
[anchorage_length_MinFS,total_length_MinFS] =
Geotextile_length(tension_allowable,required_FS,ce,transfer_efficiency,R_cri,x0_cri,y0_cri,x1,y1,y_geot
extile,apSL);

% display results
disp('-------------------critical slip surface---------------------');
disp(['location : ', num2str(location_c)]);
disp(['minimum FS            = ', num2str(MinFS)]);
disp(['reliability index      = ', num2str(Beta_MinFS)]);
disp(['probability of failure = ', num2str(Pro_MinFS)]);

% % -----------------------Sensitivity Analysis-----------------------------
```

```
Sensitivity_result = Sensitivity_analysis_cor([dis_ce dis_tfrie dis_geotextile],[rho_cf 0 0],[ce tfrie
dis_geo_parameter_1],[std_ce std_tfrie dis_geo_parameter_2],[Uc_MinFS Uf_MinFS
Ug_MinFS],Beta_MinFS);
S_mc_mFS  = Sensitivity_result(1);
S_sc_mFS  = Sensitivity_result(4);
S_mf_mFS  = Sensitivity_result(2);
S_sf_mFS  = Sensitivity_result(5);
S_mT_mFS  = Sensitivity_result(3);
S_sT_mFS  = Sensitivity_result(6);
S_rho_cf_mFS = Sensitivity_result(7);
S_rho_cT_mFS = Sensitivity_result(8);
S_rho_fT_mFS = Sensitivity_result(9);

% display sensitivity results
disp('S.Measurement of mean  S.Measurement of std   for cohesion');
disp([      S_mc_mFS              S_sc_mFS]);
disp('S.Measurement of mean  S.Measurement of std   for friction');
disp([      S_mf_mFS              S_sf_mFS]);
disp('S.Measurement of mean  S.Measurement of std   for T_allow');
disp([      S_mT_mFS              S_sT_mFS]);
disp('S.Measurement of correlation coefficient');
disp('cohesion and friction    cohesion and T    friction and T');
disp([     S_rho_cf_mFS        S_rho_cT_mFS      S_rho_fT_mFS]);

% ------------------Along most probable failure surface--------------------
MaxPf      = Pf_max;                    % maximum probability of failure
Beta_MaxPf = MinBeta;                   % minimum beta w.r.r max. pf
MaxPf_x0   = k_Pf_max;                  % iteration step of x-center
MaxPf_y0   = m_Pf_max;                  % iteration step of y-center
MaxPf_R    = n_Pf_max;                  % iteration step of radius
x0_pro     = x0(MaxPf_x0);              % x of center of slip surface
y0_pro     = y0(MaxPf_y0);              % y of center of slip surface
R_pro      = R(MaxPf_R);                % radius of slip surface
location_p = [x0_pro y0_pro R_pro];

% factor of safety along most probable failure surface
FSS_MaxPf  =
Search_for_MinFS_reinforcement(x1,x2,y1,y2,apSL,ce,tfrie,unitwS,nslice,xs1_Pf_max,xl_Pf_max,AL_Pf_
max,x0_pro,y0_pro,R_pro,y_geotextile,number_of_layers,tension_allowable);
FS_MaxPf   = FSS_MaxPf(3);

% required geotextile length
[anchorage_length_MaxPf,total_length_MaxPf] =
Geotextile_length(tension_allowable,required_FS,ce,transfer_efficiency,R_pro,x0_pro,y0_pro,x1,y1,y_g
eotextile,apSL);

% display results
disp('-------------------probablistic slip surface---------------------');
```

```
disp(['location : ', num2str(location_p)]);
disp(['maximum FS           = ', num2str(FS_MaxPf)]);
disp(['reliability index      = ', num2str(Beta_MaxPf)]);
disp(['probability of failure = ', num2str(MaxPf)]);

% ------------------------Sensitivity Analysis-----------------------------
Sensitivity_result = Sensitivity_analysis_cor([dis_ce dis_tfrie dis_geotextile],[rho_cf 0 0],[ce tfrie
dis_geo_parameter_1],[std_ce std_tfrie dis_geo_parameter_2],[Uc_MaxPf Uf_MaxPf
Ug_MaxPf],Beta_MaxPf);
S_mc_mPf  = Sensitivity_result(1);
S_sc_mPf  = Sensitivity_result(4);
S_mf_mPf  = Sensitivity_result(2);
S_sf_mPf  = Sensitivity_result(5);
S_mT_mPf  = Sensitivity_result(3);
S_sT_mPf  = Sensitivity_result(6);
S_rho_cf_mPf = Sensitivity_result(7);
S_rho_cT_mPf = Sensitivity_result(8);
S_rho_fT_mPf = Sensitivity_result(9);

% display sensitivity results
disp('S.Measurement of mean  S.Measurement of std   for cohesion');
disp([      S_mc_mPf              S_sc_mPf]);
disp('S.Measurement of mean  S.Measurement of std   for friction');
disp([      S_mf_mPf              S_sf_mPf]);
disp('S.Measurement of mean  S.Measurement of std   for T_allow');
disp([      S_mT_mPf              S_sT_mPf]);
disp('S.Measurement of correlation coefficient');
disp('cohesion and friction    cohesion and T    friction and T');
disp([     S_rho_cf_mPf        S_rho_cT_mPf      S_rho_fT_mFS]);
% --------------------Required Total Geotextile Length---------------------
anchorage_length = max(anchorage_length_MinFS,anchorage_length_MaxPf);
total_length     = max(total_length_MinFS(:),total_length_MaxPf(:));

% ------------------------Plot Geotextile Layers---------------------------
x_toe = x1; y_toe = y1;
x_intersection_slope = x_toe + (y_geotextile-y_toe)./tan(apSL);
x_end_geotextile = x_intersection_slope + total_length';
for i = 1:length(x_intersection_slope)
   xl = x_intersection_slope(i):0.2:x_end_geotextile(i);
   yl = y_geotextile(i);
   sl = line(xl,yl);
   set(sl,'Color','g','LineWidth',1,'LineStyle','x');
end
hold on;

% -------------------------------Plot Slope-------------------------------
xl1 = ((x1-x2)*2):0.05:x1;
yl1 = 0*xl1+y1;
```

```
xl2 = x1:0.01:x2;
yl2 = tan(apSL)*xl2+y1-tan(apSL)*x1;
xl3 = x2:0.01:(x2+(x2-x1));
yl3 = 0*xl3+y2;
sl1 = line(xl1,yl1);
sl2 = line(xl2,yl2);
sl3 = line(xl3,yl3);
set(sl1,'Color','black','LineWidth',2);
set(sl2,'Color','black','LineWidth',2);
set(sl3,'Color','black','LineWidth',2);
xlabel('x')
ylabel('y')
axis equal
grid on
hold on

% ----------------------------Plot center grids----------------------------
% for k = 1:nx0
%     yg = y0(1):0.1:y0(ny0);
%     xg = x0(k)+ 0*yg;
%     sg = line(xg,yg);
%     hold on;
% end
% for m = 1:ny0
%     xg = x0(1):0.1:x0(nx0);
%     yg = y0(m)+ 0*xg;
%     sg = line(xg,yg);
%     hold on;
% end

% -----------------------Plot Critial Slip Surface-----------------------
[xa0,ya0] = Plot_Potential_Slip_Surface(x0_cri,y0_cri,R_cri,xs1_FS_min,xs2_FS_min);
sp_c = plot(xa0,ya0);                          % critical slip surface
set(sp_c,'Color','red','LineWidth',2,'LineStyle','-'); hold on;
% plot(x0_cri,y0_cri,'*');                     % center of slip surface

% ------------------Plot Probabilistic Slip Surface---------------------
[xa0,ya0] = Plot_Potential_Slip_Surface(x0_pro,y0_pro,R_pro,xs1_Pf_max,xs2_Pf_max);
sp_p = plot(xa0,ya0);                          % probablistic slip surface
set(sp_p,'Color','cyan','LineWidth',3,'LineStyle',':'); hold on;
% plot(x0_pro,y0_pro,'^');                     % center of slip surface

legend([sp_c,sp_p],'critical slip surface','most probable slip surface');
% % ----------------Plot Slices for Critical Slip Surface------------------
% intv = 0.01;
% step = H/intv;
% xs1  = xs1_FS_min;
% xs2  = xs2_FS_min;
```

```
% xright  = zeros(1,nslice);
% yright  = zeros(1,nslice);
% yuright = zeros(1,nslice);
% for j = 1:nslice
%     xright(j) = xs1+j*(xs2-xs1)/nslice;
%     yright(j) = y0_cri-sqrt(R_cri^2-(xright(j)-x0_cri)^2);
%      if xright(j) < x1
%         yuright(j) = y1;
%      end
%      if (xright(j) > x1 && xright(j) < x2)
%         yuright(j) = tan(apSL)*xright(j)+y1-tan(apSL)*x1;
%      end
%      if xright(j) > x2
%         yuright(j) = y2;
%      end
%     yline      = yright(j):intv:yuright(j);
%     xline      = xright(j)+0*yline;
%     slice      = line(xline,yline);
%     set(slice,'Color','red','LineWidth',1);
%     hold on;
% end
% -----------------------------------------------------------------------------------------------------------------------------------
% -----------------------------------------------------------------------------------------------------------------------------------
function result =
MPP_cor(g_function_name,partial_g_u_name,distribution_type,correlation_coefficient,mean,std,inputs
_g)

n_variable = length(distribution_type);

% ----------------------------Initial State---------------------------------
s     = 0;
beta  = 0;
u_variable    = zeros(1,n_variable);
[L,variables] =
Transformation_correlated_variables(distribution_type,correlation_coefficient,mean,std,u_variable);
gs            = feval(g_function_name,inputs_g,variables);
dg_variable   =
feval(partial_g_u_name,inputs_g,variables)*differential_variable_cor(distribution_type,L,variables);
a_variable    = dg_variable./norm(dg_variable);

% -----------------------------Iteration----------------------------------
Step      = 2000;
tolerance = 10^(-10);
while s <= Step
   beta_previous = beta;
   g_previous    = gs;
   dg_variable_previous = dg_variable;
   a_variable_previous  = a_variable;
```

```
   u_variable_previous  = u_variable;
   s           = s+1;
   beta         = beta_previous + g_previous./norm(dg_variable_previous);
   u_variable    = -beta.*a_variable_previous;
   [L,variables] =
Transformation_correlated_variables(distribution_type,correlation_coefficient,mean,std,u_variable);
   gs           = feval(g_function_name,inputs_g,variables);
   dg_variable   =
feval(partial_g_u_name,inputs_g,variables)*differential_variable_cor(distribution_type,L,variables);
   a_variable    = dg_variable./norm(dg_variable);
   diss         = norm(double(u_variable-u_variable_previous));
   if (abs(double(gs))<tolerance && diss<tolerance)
      nn = s;
      break
   end
end
% ----------------------------------------------------------------------
format long

Pf  = 1-normcdf(beta);
g_f = double(gs);

result = [u_variable,beta,Pf,g_f];
% ------------------------------------------------------------------------------------------------------------------------------
% ------------------------------------------------------------------------------------------------------------------------------
function result = Optimization_Design(dvl,dvu,required_pf,IntCon,obj_fun,con_fun)
% -----------------Reliability-Based Optimization Design-------------------
objfun      = @(dv) feval(obj_fun,dv);
confun      = @(dv) feval(con_fun,dv);
options     = gaoptimset('PlotFcns',{@gaplotbestf},'Display','iter');
%
dv = ga(objfun,2,[],[],[],[],dvl,dvu,confun,IntCon,options);
% Posterior analysis
[fval,total_usage,usage_sec] = feval(objfun,dv);
g  = feval(confun,dv);
pf = g + required_pf;
% Display the results;
disp('---------------------RBD results------------------------');
disp(['The optimal point = ',num2str(dv)]);
disp(['The objective function = ',num2str(fval),'; primary = ',num2str(total_usage),'; secondary =
',num2str(usage_sec)]);
disp(['Probability of failure =',num2str(pf)]);
result(1,:) = dv;
result(2,1) = pf;
result(3,1) = fval;
result(4,1) = total_usage;
result(4,2) = usage_sec;
return;
```